Experimental Evidence on Advertising and Price Competition

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Abstract

This experiment tests a recent theoretical model of advertising and price competition by Du (2004). The model predicts that equilibrium prices will be lower when firms' advertising conveys the price than when it does not convey the price. In the laboratory sessions, each period human sellers make two decisions: what price to set, and whether to advertise to eliminate consumer search costs for their product. Robot buyers then follow an optimal search rule (known to all sellers) to decide which price offer (if any) to accept. The two experimental conditions are (1) advertising the price, or (2) advertising before pricing. Data from ten sessions indicate that, as predicted, firms choose more often to advertise when advertising conveys price, and prices in the second treatment are significantly higher than prices in the first treatment.

Keywords: Experiment, Advertising, Search Cost, Pricing

JEL Codes: C91, C72, D21, D43, D83, L11, L13

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I. Introduction

Conventional Bertrand Competition and the consumer search model studied by Diamond (1971) provide two extremes in predicting market outcomes. Under Bertrand Competition, sellers undercut their rivals and the market outcome is marginal cost pricing. However, Diamond shows that when consumer search costs are positive, the unique market equilibrium is monopoly pricing. The intuition is that when all buyers have positive search costs for prices, buyers will not switch to other sellers when they see a price that is slightly higher than other prices; when other sellers are charging a price below the monopoly price, choosing a slightly higher price will strictly improve one seller's profit.

One way to escape Diamond's surprising result is to introduce sources of free information to buyers. When advertising is available to sellers, they have an incentive to announce their prices in order to increase their market shares. In this way buyers observe the advertised prices for free and buyers can switch to the advertised offers without paying a search cost. The studies by Butters (1977) and Robert and Stahl (1993) show that as the advertising costs go up, the degree of price dispersion (i.e., the variance of prices) in the market will go up. When advertising is free, the market outcome goes back to marginal cost pricing.

However, the predictions in the previous advertising literature depend heavily on the assumption of the format of advertising. Both Butters and Robert & Stahl assume that sellers simultaneously make two decisions: sellers choose prices and determine the proportion of buyers that receive advertisements (ads). Price advertising is modeled in a way that presumes that prices are contained in the ads. However, this is not necessarily the most reasonable way to model advertising. As argued by Du (2004), in many real world cases, ads do not directly include price information. Nevertheless, these ads still provide a way to remove consumer search costs for sellers' prices. An advertiser may leave its location and contact method in the ads. It is very common to see examples in the Yellow Pages as the follows: "Insurance problems? Hassle free quotes! Out of area call 1-800-xxx-xxxx." For internet advertising, by clicking the banners, pop-up windows and sponsored links, potential buyers are led to the advertiser's homepage that provide information about the product details, including prices. This way, consumers can easily find out product prices by several phone calls or several clicks. Ads substantially reduce consumers' time and effort during the search process.

For these important reasons, it is plausible to think of sellers as making two decisions sequentially. First, sellers decide whether to advertise, i.e., whether to remove consumer search costs for their prices. After observing rivals' advertising choices, sellers choose their prices. Contrary to the results by Butters and Robert & Stahl, under this Advertise-then-Price assumption sellers choose not to advertise and the market outcome goes back to monopoly pricing.

The main goal of this paper is to empirically test the hypothesis that the format of advertising matters. 10 laboratory sessions were conducted at the Economic Science Laboratory (ESL), the University of Arizona. Data suggest that the prices in the markets where sellers observe their rivals' advertising choices before choosing their prices are significantly higher than the prices in the markets where sellers simultaneously make their advertising choices and choose their prices.

The paper is organized as follows. Section II discusses two market games and their predictions. Section III is the experimental design. Section IV reports the results. Section V concludes.

II. Market Games

In this section I construct two market games, the advertise-with-price game and the advertise-then-price game. The advertise-with-price game recaptures the main results by Butters and Robert & Stahl and serves as the control for the experiment. The advertise-then-price game investigates the research hypothesis that the format of advertising matters.

In both of these two games, there are N sellers, identified as 1, 2, ..., N, and each of them produces a homogeneous good with zero cost of production. Sellers' goal is to maximize profits. There are m identical buyer, who have positive search cost c and reservation value v for one unit of the good with the restriction $c < v$. The buyers can perfectly recall any past offers and buyers' goal is to maximize consumer surplus. All of the players are risk neutral. To keep the games simple, ads effectively reach the whole buyer population and sellers' advertising cost is restricted to zero.

1. Advertise-with-Price

The procedure of the game is explained as follows.

Stage 1: Each of the sellers makes a combination of two decisions: 1) choose product price P_i and 2) choose whether or not to send ads to buyers (characterized by {Ad, No Ad}). Sellers make their decisions simultaneously.

Stage 2: The buyers then observe all the ads, and make their decisions. To illustrate the game clearly, let us discuss two possible cases:

Case I: Buyers receive at least one ad. In this case buyers can take any one of the advertised offers without paying search cost, or search for the nonadvertised prices, or quit the market. If one buyer takes seller i's offer, then the buyer gets v - P_i and seller i gets P_i. If the buyer quits, all players get zero. Buyers learn one nonadvertised offer with equal chance from every search, and the cost per search is c. The search process is without replacement.

Case II: Buyers receive no ad. In this case a buyer is randomly matched with one of the sellers and observes that seller's price for free. With probability of 1/N the buyer observes seller i's price. Then the buyer decides whether to take i's offer without paying the search cost, or continue to search for the other offers (without replacement), or quit. Again, the cost per search is c.

I focus on the price offers that do not give the player losses if accepted, $P_i \in [0, v]$, i=1, ..., N. I assume that the buyers' tie breaking rule is to randomly take one of the lowest observed prices (i.e., if there are n tying observed prices, the buyer takes one of these prices with probability of 1/n). Then from the game, we have the following observation.

Observation 1 In Nash equilibrium of the advertise-with-price game, at least one seller i chooses (P_i = 0, Ad); the buyers' strategy is to take the lowest observed offer.

All proofs are collected in Appendix A.

Observation 1 indicates that the market outcome becomes marginal cost pricing when the pricing choice and the advertising choice are made simultaneously by each seller. The intuition is that when other sellers are charging a price above the marginal cost, one seller has an incentive to advertise a slightly lower price in order to take the whole market. This result is consistent with the previous literature.

2. Advertise-then-Price

The procedure of the game is explained as follows.

Pre-stage: sellers simultaneously decide whether to send ads to buyers (characterized by {Ad, No Ad}).

Stage 1: Pre-stage decisions by sellers are realized by all sellers. Sellers simultaneously choose their prices $P_1, P_2, ..., P_N$.

Stage 2: Now it is buyers' turn to move. The description of the buyers' moves is exactly the same as that of the advertise-with-price game.

Since advertising choices are realized before sellers choose prices, we can break the original game into an "advertising game" and a "pricing game": given every seller's advertising choice, there is a corresponding separable pricing subgame. What we need to do is to discuss three possible cases. Again, let us focus on the price offers $P_i \in [0, v]$, $i=1, ..., N$.

Case I: All of the sellers choose 'not to advertise.'

Let us call this pricing subgame the "search cost game." Here I use *search equilibrium*, the standard solution concept in consumer search and price dispersion literature which is developed by Burdett and Judd (1983), as the solution to this pricing subgame.

In order to define search equilibrium, one needs to first specify the buyers' optimal cutoff rule. Suppose one buyer observes k price offers. There are still $N - k$ prices unknown to that buyer. Without loss of generality, suppose that $P_1, ..., P_k$ are observed by that buyer but

P_{k+1}, ..., P_N are not observed. Let the lowest observed price be z. Then at the k^{th} observed price, the buyer has optimal cutoff price $r_k = \min(v, \frac{1}{N-k}\sum_{j=k+1}^{N} P_j + c)$. When z is less than or equal to r_k, the buyer accepts z. Otherwise the buyer searches for a new price. The search equilibrium is defined as follows.

Definition The strategy profile (P_1^*, ..., P_N^*, (r_1^*,…, r_{N-1}^*)) is a *search equilibrium* if:

(1) For sellers, $\pi_i (P_i^*, P_{-i}^*, (r_1^*, ..., r_{N-1}^*)) \geq \pi_i (P_i, P_{-i}^*, (r_1^*, ..., r_{N-1}^*))$ for $\forall P_i$, $i = 1, ..., N$. π_i is seller i's payoff.

(2) For the buyers, r_1^*, ..., r_{N-1}^* are the optimal cutoff prices at the 1st, 2nd, ..., (N-1)th observed price, respectively.

Notice that according to the definition, search equilibrium is a refinement of Nash equilibrium. Then we have the following observation in this pricing subgame.

Observation 2 In the search cost game (a pricing subgame of the advertise-then-price game), the unique search equilibrium is ($P_1^* =...= P_N^* = v$, $r_1^* =...= r_{N-1}^* = v$). Each of the sellers gets expected payoff v/N, and buyers get zero payoff.

Case II: One seller chooses 'to advertise,' but all other sellers choose 'not to advertise.'

Let us call this pricing subgame the "one-ad pricing game." Without loss of generality, suppose seller 1 chooses to advertise, all of the other sellers choose not to advertise. Then we get the third observation.

Observation 3 In the one-ad pricing game (a pricing subgame of the advertise-then-price game), given seller 1 is the advertiser, the unique search equilibrium is ($P_1^*=c$, $P_2^*=...=P_N^*=0$, $r_1^*=...=r_{N-1}^*= c$). The advertiser gets payoff c, the nonadvertisers get zero payoff.

Case III: At least two of the sellers choose 'to advertise.'

The seller who advertises its price always has an incentive to undercut the other advertisers' prices in order to take the whole market. There is Bertrand competition among the advertisers. Finally, the advertisers set prices at zero and the nonadvertisers cannot make a sale. In Nash equilibrium, all sellers get payoff zero and the buyers get payoff v; the buyers take the lowest advertised price.

Now I have discussed all possible pricing subgames. According to the discussion, every pricing subgame has a unique market outcome (i.e., the payoffs to sellers and buyers are unique). Then one can substitute every pricing subgame by its market outcome, and use backward induction to solve for the advertising game. Observation 4 describes the solutions in the advertising game.

Observation 4 In the simplified advertising game of the advertise-then-price game, when $0 < c < \frac{v}{N}$, strategy 'not to advertise' weakly dominates 'to advertise' for all sellers and the strategy profile that all of the sellers choose 'not to advertise' is the unique weakly dominant strategy equilibrium.

Observation 4 says that when consumer search cost is relatively low (namely $0 < c < \frac{v}{N}$), monopoly pricing is more likely to be the market outcome in the advertise-then-price game. This result contradicts the findings in the previous price advertising literature. The intuition is that when sellers can observe their rivals' advertising choices before pricing, backward induction may reduce the possibility of entering the pricing subgames that have low equilibrium prices.

III. Experimental Design

In the laboratory sessions, all participants act as sellers. The buyers are computer-simulated and these automated buyers follow the equilibrium strategies described in the previous section.

There are several important reasons to use automated buyers instead of human buyers. First, my experiments incorporate a finite number of buyers. As argued by Coursey, Isaac and Smith (1984), incorporating a finite number of human buyers "could leave open the possibility that the competitive discipline of the markets is due not directly to contesting by sellers but rather to the actual (or merely anticipated) strategic withholding of demand by buyers." Using automated buyers allows me to control for strategic withholding of demand and gives the theory best chance to survive. If one observes marginal cost pricing in laboratory sessions, this must due directly to competition among sellers rather than buyers' market power. Moreover, how the buyers' market power influences market prices is not the research question of this paper. Secondly, when buyers' population is relatively large, the assumption that buyers reveal demand is quite realistic. It is hard to imagine that buyers strategically withhold demand when they shop in grocery stores and bookstores. Finally, using automated buyers substantially reduces the payments to subjects.

The main goal of this paper is to answer the question of how the format of advertising matters. There are two games in the experiments, the advertise-then-price game and the advertise-with-price game.

In the advertise-then-price games, sellers first simultaneously decide whether to advertise their prices: they click the button "Reveal" if they decide to advertise, and they click "Not to Reveal" if they decide not to advertise. Then the sellers will observe their rivals' advertising choices ("Reveal" or "Not to Reveal") and will enter their prices in the given text box. In the advertise-with-price games, all sellers simultaneously make their advertising and pricing choices.

In both advertise-then-price and advertise-with-price games, advertising is free for sellers. The buyer search cost is fixed at 30 cents and the number of sellers in the market is fixed at three. All sellers have zero costs and have no capacity constraint. The buyers' reservation value is \$2.00. According to the market games in the previous section, in the advertise-then-price games, 'all sellers choose not to advertise' is the weakly dominant strategy equilibrium in the simplified advertising game and the equilibrium price is \$2.00. In the advertise-with-price games, 'all sellers choose to advertise, and sellers choose prices at zero' is the equilibrium prediction.

The number of automated buyers in the market is also three. Each automated buyer demands one unit of the good. The automated buyer's shopping rule, which follows the equilibrium path, is known by all sellers.

10 sessions were conducted at the Economic Science Laboratory (ESL), the University of Arizona, from November 2003 to January 2004. The experimental software was written in Visual Basic 6.0. 5 sessions are advertise-then-price games (identified as ATP1, ATP2,…,ATP5) and the other 5 sessions are advertise-with-price games (identified as AWP1, AWP2,…,AWP5). In each session, 6 human subjects are recruited as sellers and 20 trading periods are scheduled. At the beginning of every trading period, 6 sellers are randomly divided into two markets, 3 sellers in each market. At the end of every trading period, sellers review their own profit or loss and observe the choices of the other sellers from the same market. Subjects were randomly chosen from the ESL experiment recruiter database. Those who registered in the database must have a valid University of Arizona student ID card. Each subject participated in only one session. The experimental treatments were implemented across-subjects; that is, different subjects participated each of the two market games.

Each subject was paid $5 show-up fee plus the earnings during the experiment. The earnings during the experiment were recorded in experimental dollars. The experimental dollars were convertible to USD at the rate of 0.5 USD per experimental dollar in the advertise-then-price games, and at the rate of 1 USD per experimental dollar in the advertise-with-price games. The average payment in the advertise-then-price games was $19.95, and the average payment in the advertise-with-price games was $11.76. The length of the advertise-with-price games was about 45 minutes. The length of the advertise-then-price games was about 1 hour and 20 minutes and the task was moderately more complicated.

Since there are only six subjects in each session, the common history of plays cannot be fully avoided. In order to control for statistical dependence, each session of each market game is treated as one independent observation.

IV. Results

Sellers need to make two decisions in the market games. They need to decide whether to advertise and they need to choose their prices. These two decisions are separately analyzed as follows.

1. Sellers' Advertising Choices

The proportions of the advertising choices in advertise-with-price games are reported in table 1. From table 1 we can see that in each session, the proportion of "*all of the sellers decide to advertise*" ('all advertise' in table 1) dominates the proportion of "*All of the sellers decide not to advertise*" ('no advertise' in table 1) and dominates the proportion of "*some of the sellers decide to advertise, but the other sellers decide not to advertise.*" ('other advertising choices' in table 1). We can see from the table that the proportions of 'no advertise' in all 5 advertise-with-price sessions are zero. Table 2 shows that both means test and Kolmogorov-Smirnov test suggest that the proportion of 'all advertise' is greater than 0.5 in advertise-with-price games. The test statistics are significant at 1% level.

It is shown in table 3 that in advertise-then-price games the proportion of 'no advertise' dominates the proportion of 'all advertise.' However, there are a lot of noises. In session ATP2 and session ATP4, the proportion of 'other advertising choices' is higher than the proportion of 'no advertise.' Overall, the proportion of 'no advertise' is much higher than the proportion of choices of 'other advertising choices.' Table 4 shows the tests on the proportion of 'no advertise' in advertise-then-price games. The mean test suggests that the proportion of 'no advertise' is greater than 0.5, though the test statistics is marginal significant (p-value is 0.06). The

Kolmogorov-Smirnov test suggests that we cannot reject the null hypothesis that the proportion of 'no advertise' is equal to 0.5.

Table 5 reports the tests on the treatment effect on advertising. The test results show that the treatment effect is highly significant. Comparing the advertise-with-price games and the advertise-then-price games, the proportion of 'all advertise' decreases from 0.845 to 0.03, and the proportion of 'no advertise' increases from 0 to 0.755. Both means test statistics and Kolmogorov-Smirnov test statistics are significant at 1% level.

2. *Sellers' Pricing Decisions*

The mean transaction prices in advertise-with-price games are shown in figure 1. The prices start to be high and decrease over periods. By the end of the sessions, the prices converge to zero. We find strong evidence to support marginal cost pricing in advertise-with-price games.

The mean transaction prices in advertise-then-price games are shown in figure 2. In session ATP1, ATP3 and ATP5, subjects choose the weakly dominant strategy 'not to advertise' at the very beginning, and the prices are maintained close to buyers' reservation price \$2.00. In ATP2, the prices stay low in the first 10 periods. From period 11 to period 20, subjects start to figure out that 'not to advertise' gives them better payoff and the prices start to clime. By the end of the session, the market prices are maintained at \$2.00. There are a lot of fluctuations in ATP4.

The comparison of mean transaction prices between advertise-with-price games and advertise-then-price games is shown in table 6. We can see that mean transaction prices in five advertising-then-price sessions are all higher than mean transaction prices in advertising-with-price sessions. The tests on mean transaction prices are reported in table 7. The treatment effect on mean transaction prices is highly significant. By comparing the advertise-with-price games and the advertise-then-price games, the mean transaction prices increases from \$0.338 to \$1.495. All three test statistics (means test, Kolmogorov-Smirnov test and Mann-Whitney test) are significant at 1% level.

V. Concluding Remarks

This paper provides an experimental test of a recent theoretical model of advertising and price competition by Du (2004). In a controlled laboratory environment, a market for a homogeneous good is created in which three human sellers compete to sell to three robot buyers. Each seller makes two decisions: what price to set, and whether to advertise to eliminate consumer search costs for their product. Each robot buyer, who is constrained to buy at most one unit of the commodity, then accepts an observed price, drops out, or pays a search cost to find an unobserved price according to the optimal search rule known by all sellers. The two experimental conditions are (1) advertising the price, or (2) advertising before pricing. As stated previously, theory predicts that equilibrium prices will be lower when firms' advertising conveys the price than when it does not convey the price.

Data from ten laboratory sessions indicate that, as predicted, firms choose more often to advertise when advertising conveys price, and prices in the second treatment are significantly higher than prices in the first treatment. The empirical evidence from this paper strongly supports the hypothesis that the market price is sensitive to the nature of the format of advertising.

One possible implication of this study is the internet advertising. The form of internet advertising is 'advertising-then-price': most online stores use the banners, pop-up windows and sponsored links to advertise their existence, instead of directly advertising prices. As discussed in the introduction, these ads still provide a way to remove consumer search costs for sellers' prices. Conventional theory of information economics predicts that the source of free information reduces both price and price dispersion (Stigler 1961). However, some previous empirical studies suggest that online prices are higher than prices in conventional bricks-and-mortar stores, given the truth that consumers are able to browse the online prices at home and find what they are looking for without incurring transportation cost (Bailey 1998 and Lee 1997). This study provides one possible explaination to aleviate this puzzle.

Table 1 – Advertising Choices: Advertise-with-Price Games

	Proportion of 'No Advertise'	Proportion of 'All Advertise'	Proportion of Other Advertising Choices
AWP1	0	0.9	0.1
AWP2	0	0.95	0.05
AWP3	0	0.875	0.125
AWP4	0	0.65	0.35
AWP5	0	0.85	0.15
Average Proportion	0	0.845	0.155

Table 2: Tests on Average Proportion of 'All Advertise' (Advertise-with-Price Games)

Average Proportion of 'All Advertise' (P_A)	Alternative Hypothesis	Means Test (*t*)	Kolmogorov-Smirnov Test (*D*)
0.845	$P_A > 0.5$	6.70 (p=0.0013)	1 ($p < 0.01$)

Table 3 – Advertising Choices: Advertise-then-Price Games

	Proportion of 'No Advertise'	Proportion of 'All Advertise'	Proportion of Other Advertising Choices
ATP1	1	0	0
ATP2	0.425	0.1	0.475
ATP3	0.9	0	0.1
ATP4	0.45	0.05	0.5
ATP5	1	0	0
Average Proportion	0.755	0.03	0.215

Table 4: Tests on Average Proportion of 'No Advertise' (Advertise-then-Price Games)

Average Proportion of 'No Advertise' (P_N)	Alternative Hypothesis	Means Test (*t*)	Kolmogorov-Smirnov Test (*D*)
0.755	$P_N > 0.5$	1.95 (p=0.06)	0.6 ($p > 0.05$)

Table 5: Tests on Average Proportions of Advertising Choices (Advertise-with-Price Games vs. Advertise-then-Price Games)

	Advertise-with-Price Games	Advertise-then-Price Games	Alternative Hypothesis	Means Test (*t*)	Kolmogorov-Smirnov Test (*D*)
Proportion of 'All Advertise' (P_A)	0.845	0.03	$P_A^{ATP} < P_A^{AWP}$	14.76 (p<0.0001)	1 ($p < 0.01$)
Proportion of 'No Advertise' (P_N)	0	0.755	$P_N^{AWP} < P_N^{ATP}$	5.77 (p=0.0022)	1 ($p < 0.01$)

Table 6: Mean Transaction Prices

	Mean Transaction Price		Mean Transaction Price
ATP1	1.944	AWP1	0.347
ATP2	0.877	AWP2	0.477
ATP3	1.696	AWP3	0.187
ATP4	0.967	AWP4	0.37
ATP5	1.993	AWP5	0.309
Advertise-then-Price Games	1.495	Advertise-with-Price Games	0.338

Table 7: Tests on Mean Transaction Prices (Advertise-with-Price Games vs. Advertise-then-Price Games)

Advertise-with-Price Games(P^{AWP})	Advertise-then-Price Games(P^{ATP})	Alternative Hypothesis	Means Test	Kolmogorov-Smirnov Test (D)	Mann-Whitney Test (U)
0.338	1.495	$P^{AWP} < P^{ATP}$	4.74 (p=0.005)	1 ($p < 0.01$)	0 ($p < 0.01$)

Figure 1 – Mean Transaction Price: Advertise-with-Price Games

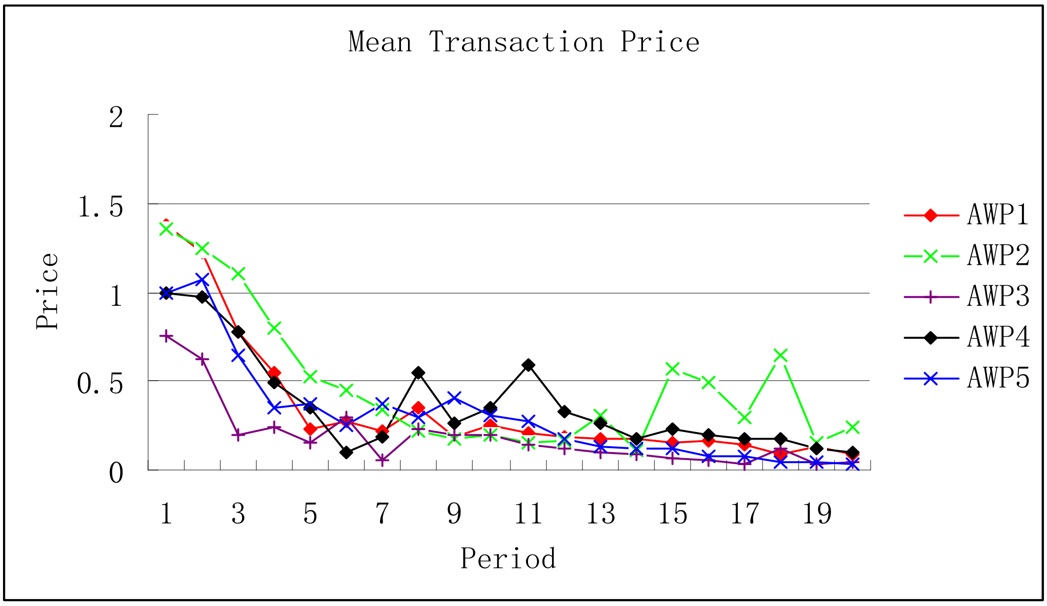
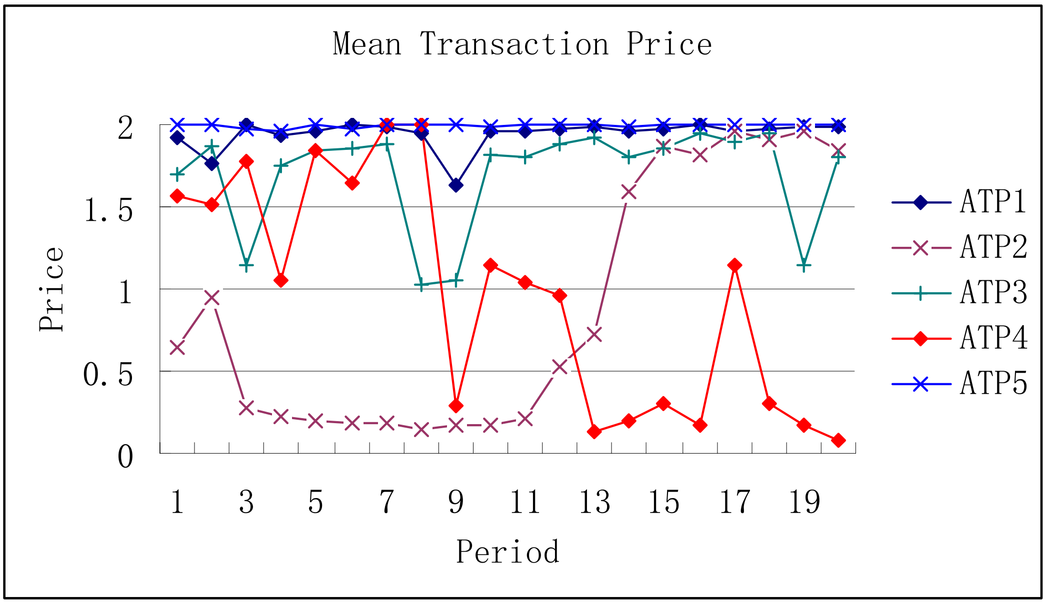


Figure 2 – Mean Transaction Price: Advertise-then-Price Games



References

Bailey, J., 1998. *Intermediation and Electronic Markets: Aggregation and Pricing in Internet Commerce*. Ph.D Thesis, Department of Electrical Engineering and Computer Science, MIT, May 20, 1998.

Burdett, K., and K. Judd, 1983. Equilibrium Price Dispersion. *Econometrica* 51: 955-969

Butters, G. R., 1977. Equilibrium Distributions of Sales and Advertising Prices. *Review of Economic Studies* 44: 465-91

Capra, M., R. Gomez, C. Holt and J. Goeree, 2002. Learning and Noisy Equilibrium Behavior in an Experimental Study of Imperfect Price Competition. forthcoming in the *International Economic Review* 43: 613 - 636.

Cason, T., and D. Friedman, 2003. Buyer Search and Price Dispersion: A Laboratory Study. *Journal of Economic Theory* 112: 232-260

Coursey, D., R. M. Isaac and V. L. Smith, 1984. Natural Monopoly and Contested Markets: Some Experimental Results. *Journal of Law and Economics* 27: 91-113

Diamond, P., 1971. A Model of Price Adjustment. *Journal of Economic Theory* 3: 158-168

Du, N., 2004. On Advertising and Price Competition. Working Paper.

Friedman, D., 1989. Producers' Markets: A Model of Oligopoly with Sales Costs.

Journal Economic Behavior and Organization 11: 381-398

Harrison, G., and M. McKee, 1985. Monopoly Behavior, Decentralized Regulation, and Contestable Markets: An Experimental Evaluation. *The Rand Journal of Economics* 16: 51-69

Lee, H. G., 1997. Do Electronic Marketplaces Lower the Price of Goods? *Communications of the ACM* 41: 73-80

Morgan, J., H. Orzen and M. Sefton, 2003. A Laboratory Study of Advertising and Price Competition. *European Economic Review* 50: 323-347

Morgan, J., H. Orzen and M. Sefton, 2006. An Experimental Study of Price Dispersion. *Games and Economic Behavior* 54: 134-158

Robert, J., and D. O. Stahl, II, 1993. Informative Price Advertising in a Sequential Search Model *Econometrica* 61: 657-86

Stigler, G. J., 1961. The Economics of Information. *Journal of Political Economy* 69: 213-225

Appendix: MATHEMATICAL DETAILS

Observation 1 In Nash equilibrium of the advertise-with-price game, at least one seller i chooses ($P_i = 0$, Ad); the buyers' strategy is to take the lowest observed offer.

Proof:

We first prove that at least one seller chooses price at zero. Suppose all sellers choose positive prices in Nash equilibrium. Let $p = \min \{P_2^*, P_3^*, ..., P_N^*\}$. Then $0<P_1<p$ cannot be seller 1's price in Nash equilibrium. Advertising a price $\tilde{p}$, where $P_1 < \tilde{p} < p$, gives seller 1 strictly better payoff. $P_1 = p$ cannot be seller 1's price in Nash equilibrium. According to buyers' tie breaking rule, advertising a price $\tilde{p}$, where $\tilde{p} = 0.9p$, gives seller 1 strictly better payoff. Choosing a price $P_1 > p$ gives seller 1 zero payoff. Therefore, $P_1 > p$ cannot be seller 1's price in Nash equilibrium. This contradicts the assumption that all sellers choose positive prices in Nash equilibrium.

Secondly, we show that at least one seller who chooses price at zero must advertise that price. Suppose in Nash equilibrium all sellers who choose price at zero do not advertise that price. Without loss of generality, suppose in Nash equilibrium seller 1 chooses $P_1^* = 0$ and seller 1 chooses not to advertise. Then advertising a price $\tilde{p}$ where $0< \tilde{p} < c$ gives seller 1 positive payoff. This contradicts the assumption that $P_1^* = 0$ is in Nash equilibrium.

Given at least one seller i chooses ($P_i = 0$, Ad) in Nash equilibrium, the buyers' best response must be taking the lowest observed offer.

Q. E. D.

Observation 2 In the search cost game (a pricing subgame of the advertise-then-price game), the unique search equilibrium is ($P_1^* =...= P_N^* = v$, $r_1^* =...= r_{N-1}^* = v$). Each of the sellers gets expected payoff v/N, and buyers get zero payoff.

Proof:

Since the sellers are in symmetric positions and the buyers cannot identify the identity of sellers, the buyers' cutoff prices must be symmetric. Therefore, the search equilibrium must be of the form ($P_1^* =...= P_N^* = r$, $r_1^* =...= r_{N-1}^* = r$). Since r is the optimal cutoff price for buyers, it must be true that $r = \min\{v, r + c\}$. The only solution to this equation is $r = v$.

Q. E. D.

Observation 3 In the one-ad pricing game (a pricing subgame of the advertise-then-price game), given seller 1 is the advertiser, the unique search equilibrium is ($P_1^*=c$, $P_2^*=...=P_N^*=0$, $r_1^*=...=r_{N-1}^*= c$). The advertiser gets payoff c, the nonadvertisers get zero payoff.

Proof:

It can be easily verified that ($P_1^*=c$, $P_2^*=...=P_N^*=0$, $r_1^*=...=r_{N-1}^*=c$) is a search equilibrium. We need to show the uniqueness of this search equilibrium.

First, $P_1 < c$ cannot be part of the equilibrium. When $P_1 < c$, buyers always take seller 1's price according to their optimal cutoff rule. However, choosing a price $\tilde{p}$, where $P_1 < \tilde{p} < c$ gives seller 1 strictly better payoff.

$P_1 > c$ cannot be part of the equilibrium. Given $P_1 > c$ and buyers' optimal cutoff rule, sellers 2, 3, ..., N will choose prices less than $P_1 - c$ to induce search. This gives seller 1 payoff zero.

Finally, we need to discuss the case when $P_1 = c$. max $\{P_2, P_3, ..., P_N\}>0$ cannot be part of the search equilibrium. Given max $\{P_2, P_3, ..., P_N\}>0$, buyers have optimal cutoff price $r_1 = \min(v, \frac{1}{N-1}\sum_{j=2}^{N} P_j + c) > c$. Then choosing a price $\tilde{p}$, where $c < \tilde{p} < r_1$ gives seller 1 strictly better payoff.

Q. E. D.

Observation 4 In the simplified advertising game of the advertise-then-price game, when $0 < c < \frac{v}{N}$, strategy 'not to advertise' weakly dominates 'to advertise' for all sellers and the strategy profile that all of the sellers choose 'not to advertise' is the unique weakly dominant strategy equilibrium.

Proof:

Observation 4 directly follows from the discussion in section III.

Q. E. D.